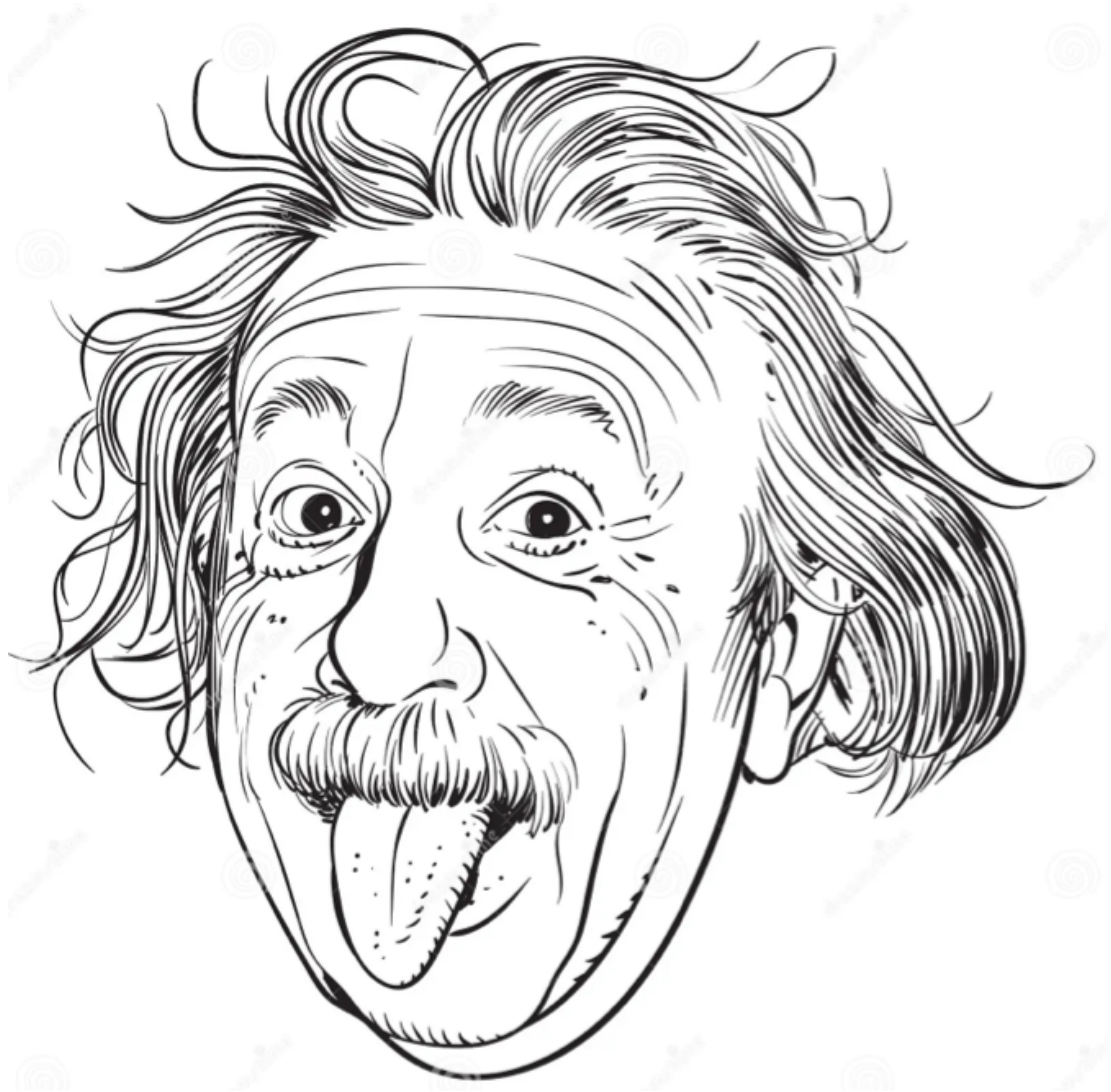
Philosophy of Physics

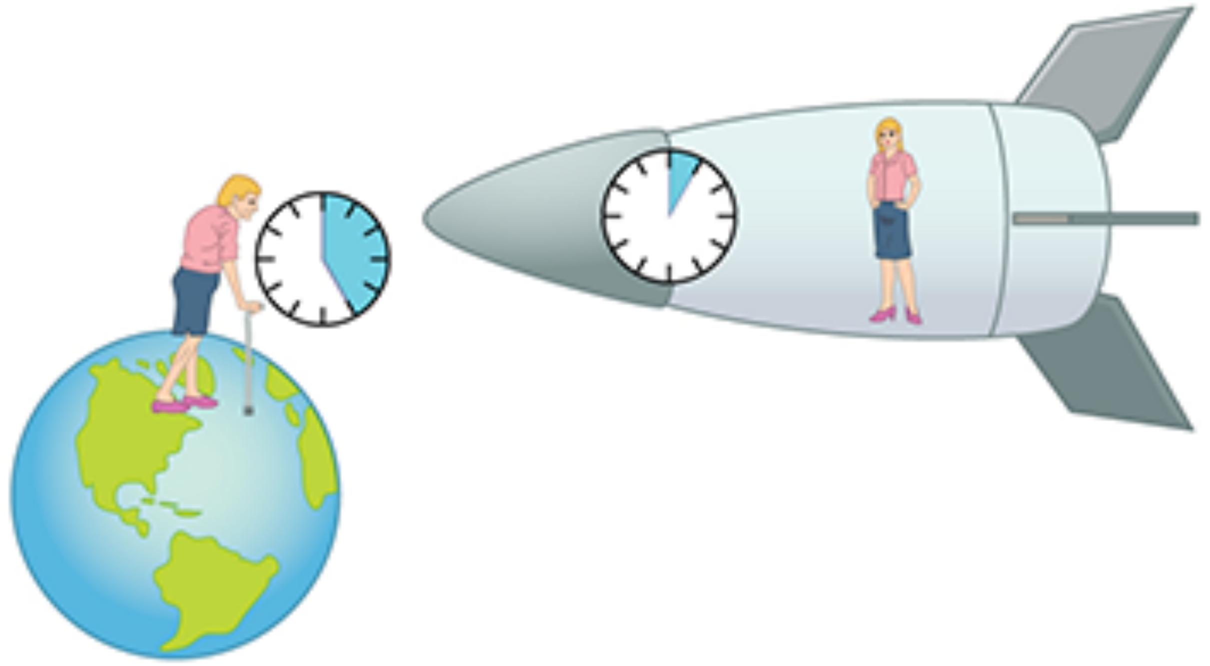
-

Lec. 5 - General Relativity



Recap: Relativity in special relativity

1. **Time and Length Dependence:**
   - No universal planes of equal times: simultaneity depends on the frame
   - Length contraction: objects appear shorter to a moving observer
   - Time dilation: spatial displacement lowers the proper time of a clock

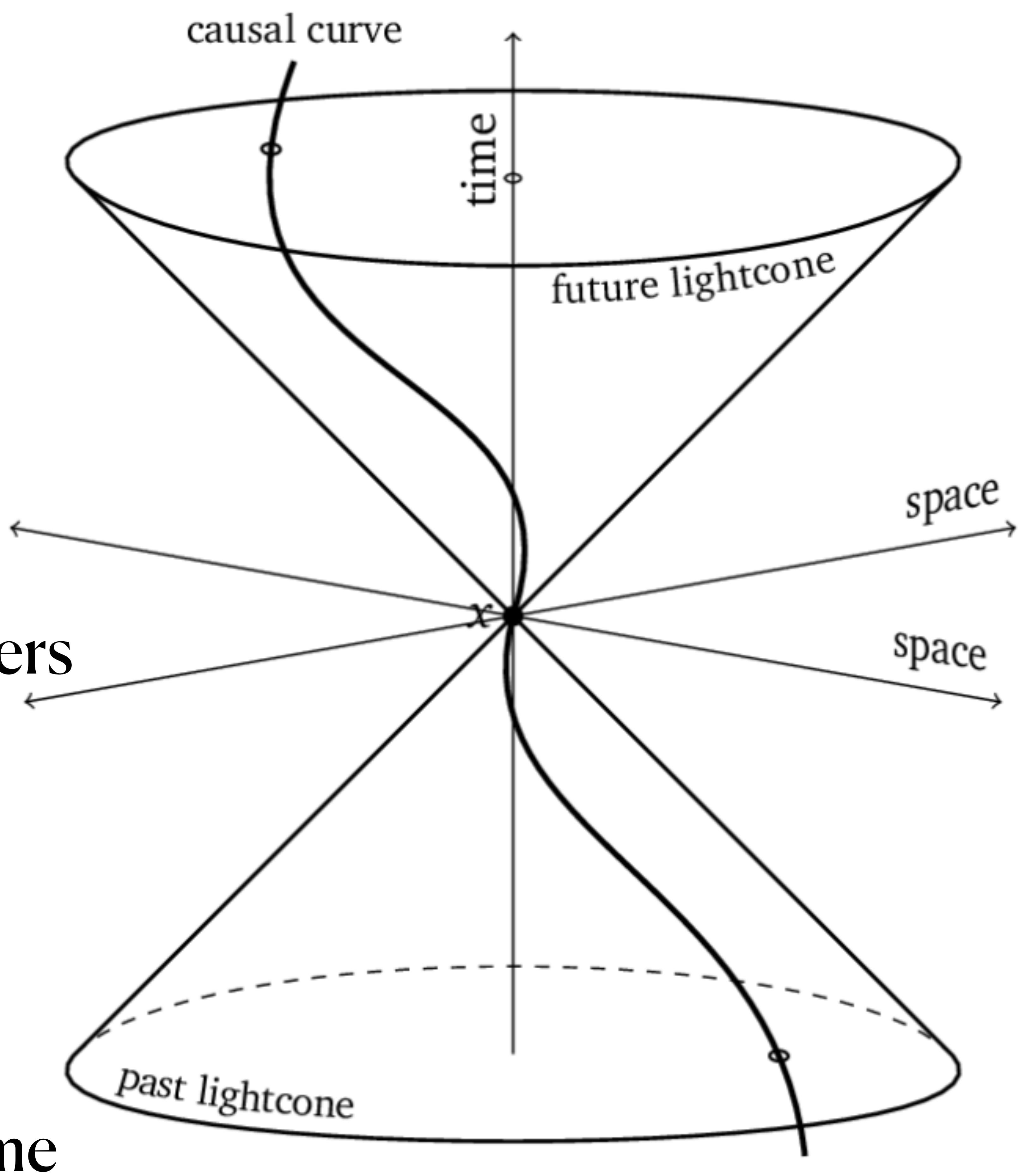


2. **Mass and Velocity:**
   - Mass increases with velocity; objects cannot reach the speed of light
   - Mass-energy equivalence

$$E=mc^2$$

Absoluteness in special relavitiy

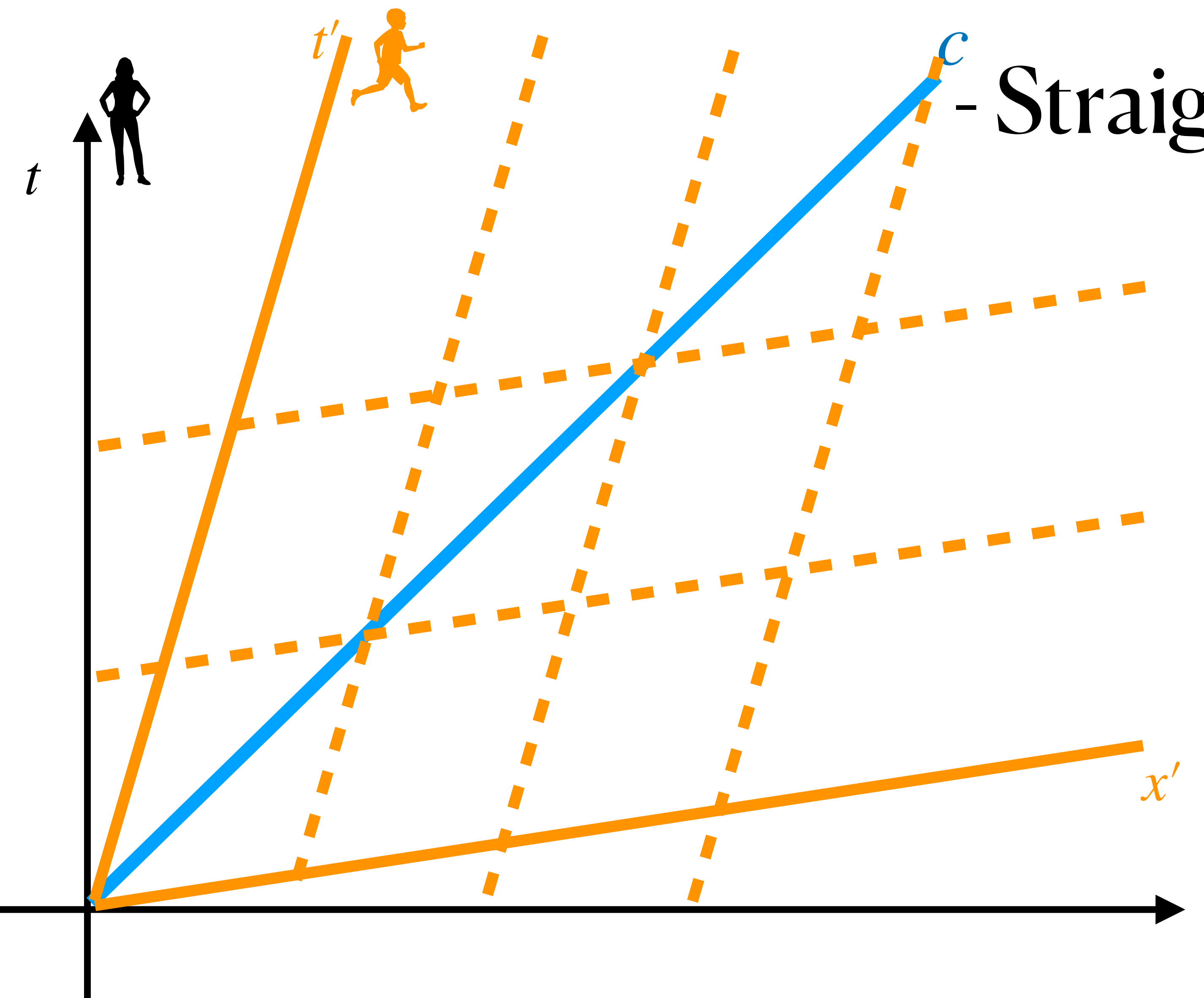
1. **Invariant quantities:**
   - Proper distance/time is the same for all observers
   - The speed of light is observer-independent (always?)
2. **Space-time curves:**
   - Spacelike, timelike, and lightlike curves remain unchanged for all observers
3. **Conserved quantities:**
   - Electric charge is the same for all observers, etc
   - Rest mass is a universal characteristic within an object's own inertial frame



Acceleration and non-inertial frames

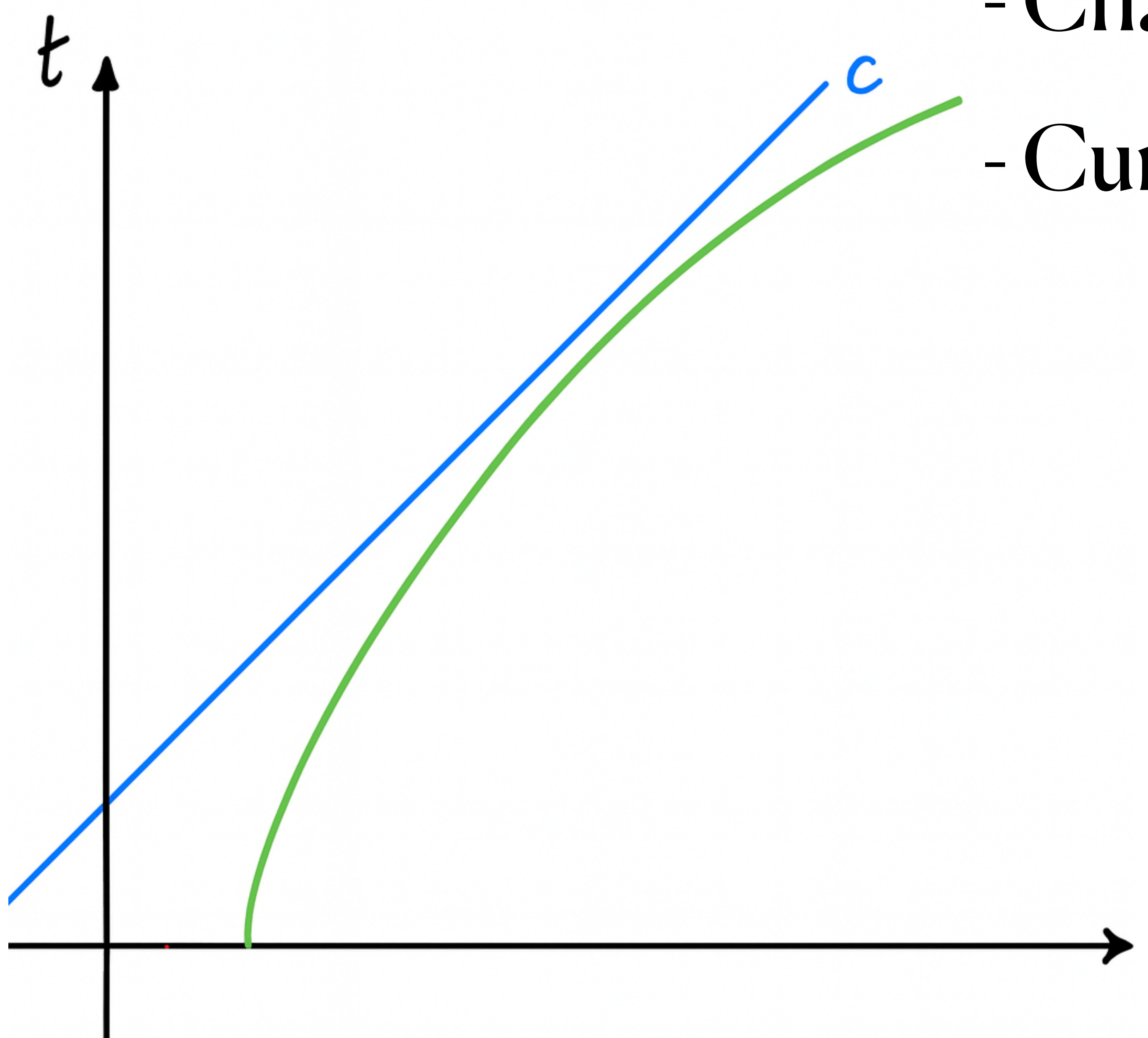
S.R. (Inertial Frames)

- Laws of physics are the same (ignoring gravity)
- Speed of light is constant
- When viewed from inertial frames:
  - Constant velocity
  - Straight-line coordinates



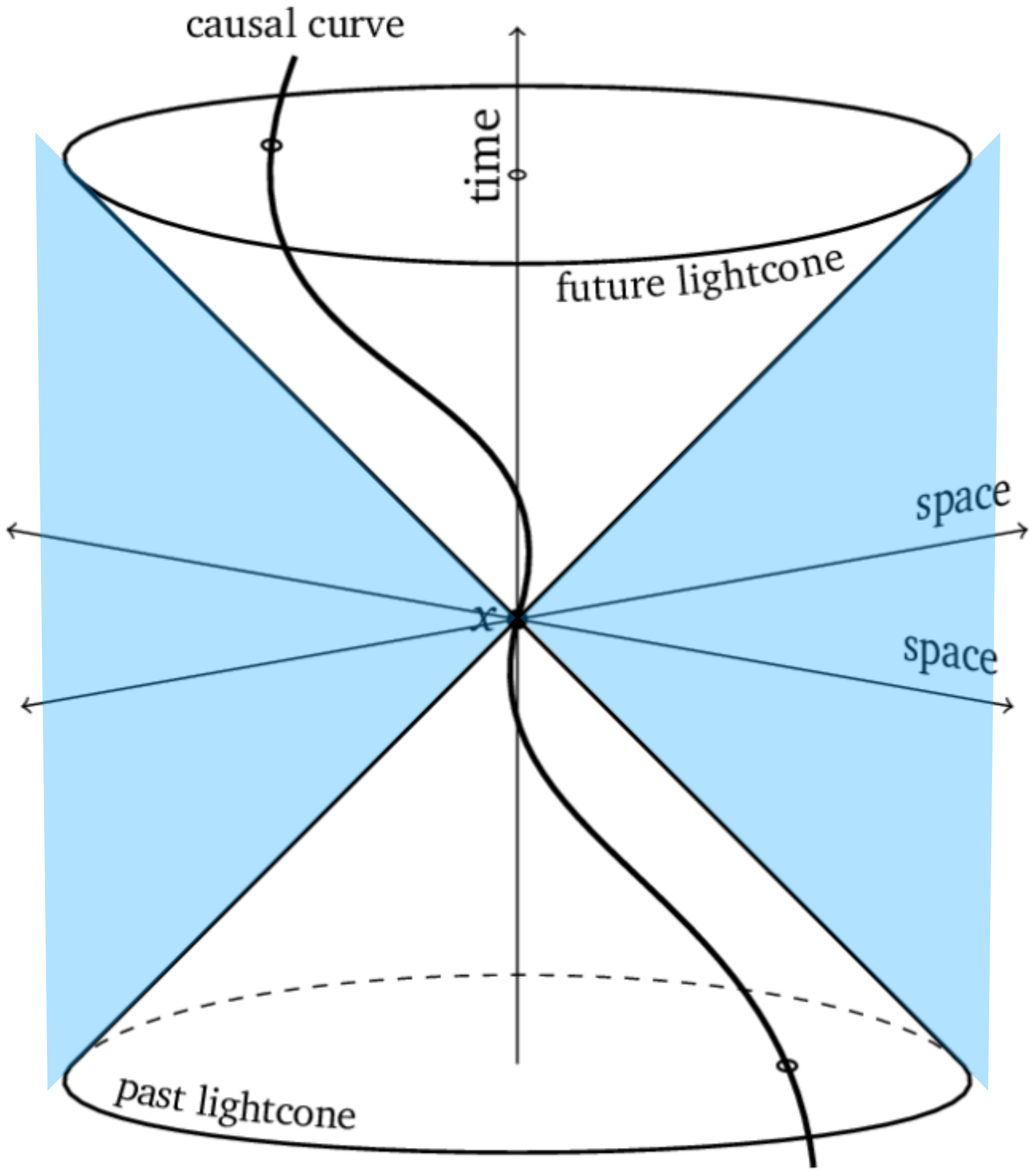
S.R. (Non-Inertial Frames)

- Extra fictitious forces exist
- Speed of light depends on the frame
- When viewed from inertial frames:
  - Changing velocity (acceleration)
  - Curvilinear coordinates



The debate from a relativistic perspective

1. **Special Relativity's Implication:**
   - No absolute meaning of equal time; it depends on the chosen inertial frame
2. **Challenges to Philosophical Positions:**
   - **Presentism:** Treats equal time as the boundary of existence
   - **Growing Universe:** Expands existence progressively over time
3. **Core Issue:**
   - They require an absolute surface of equal time to separate past and future
   - Special relativity contradicts this requirement
4. **Conclusion:**
   - Special relativity aligns with an eternalist view, where all times are equally real



The role of mathematis in Newtonian physics

Two classes ago, we ended with the mathematical aporia. On the other side, though:

1. **Auxiliary Role of Mathematics**
    - Makes observations precise
    - Represents regularities in nature
    - Provides a basis for exact prediction
2. **Newtonian Gravity and Intuition:**
    - Builds on intuition about objects in 3D space
    - Explains observed planetary motion (2D) using equations in 3D flat space
    - Relates to everyday concepts like position, velocity, and acceleration

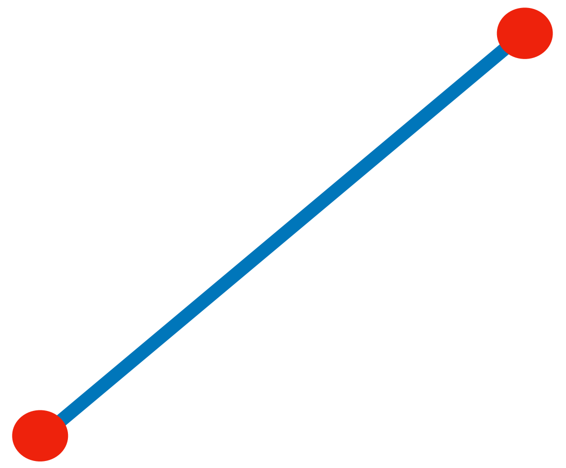
Question:

What is the role of intuition in science?

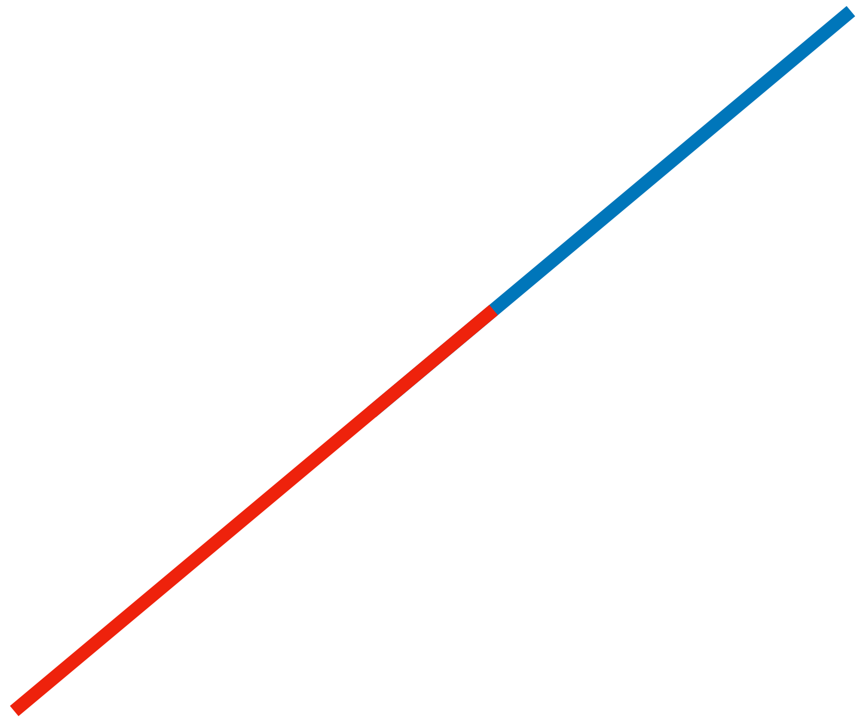
Euclid and the Parallel Axiom

5 postulates for plane geometry:

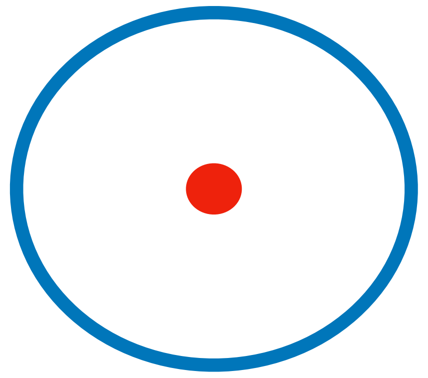
1. To draw a straight line from any point to any point



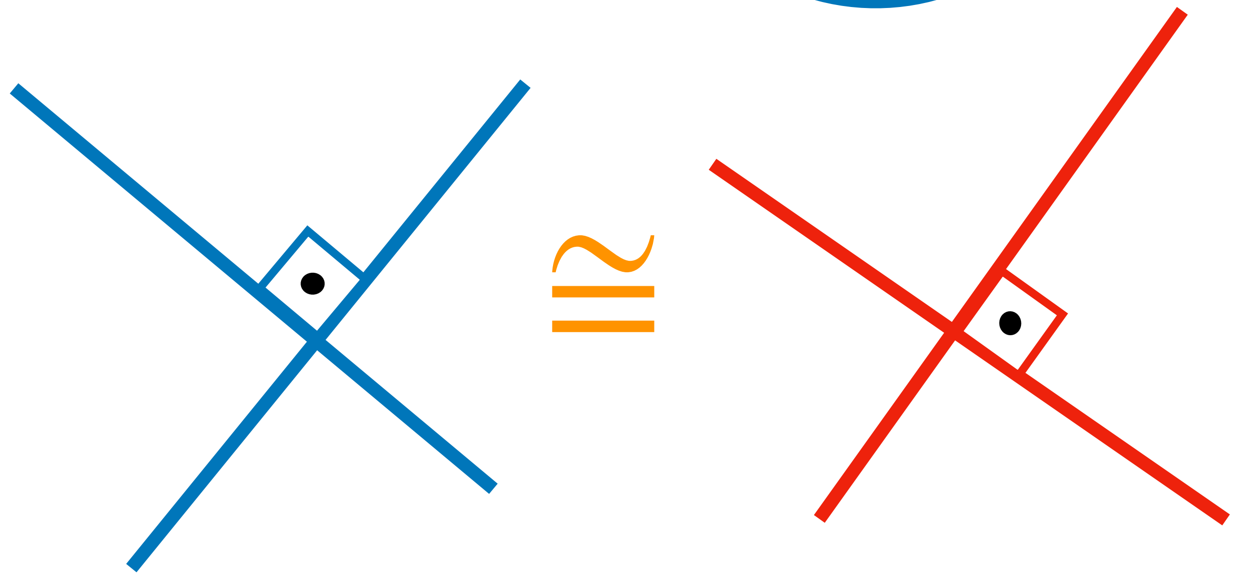
2. Any straight line segment can be extended indefinitely in a straight line



3. To describe any circle with any center and distance (radius)

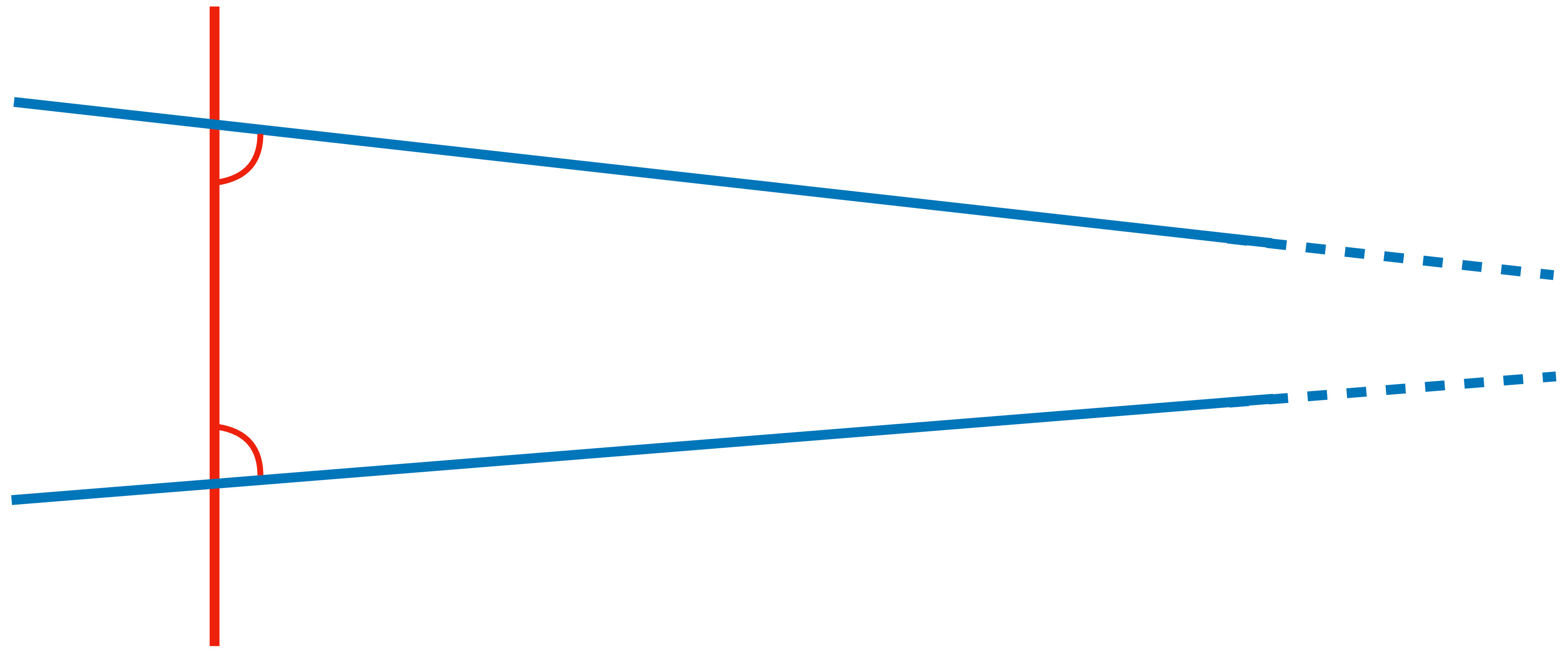


4. That all right angles are equal to one another

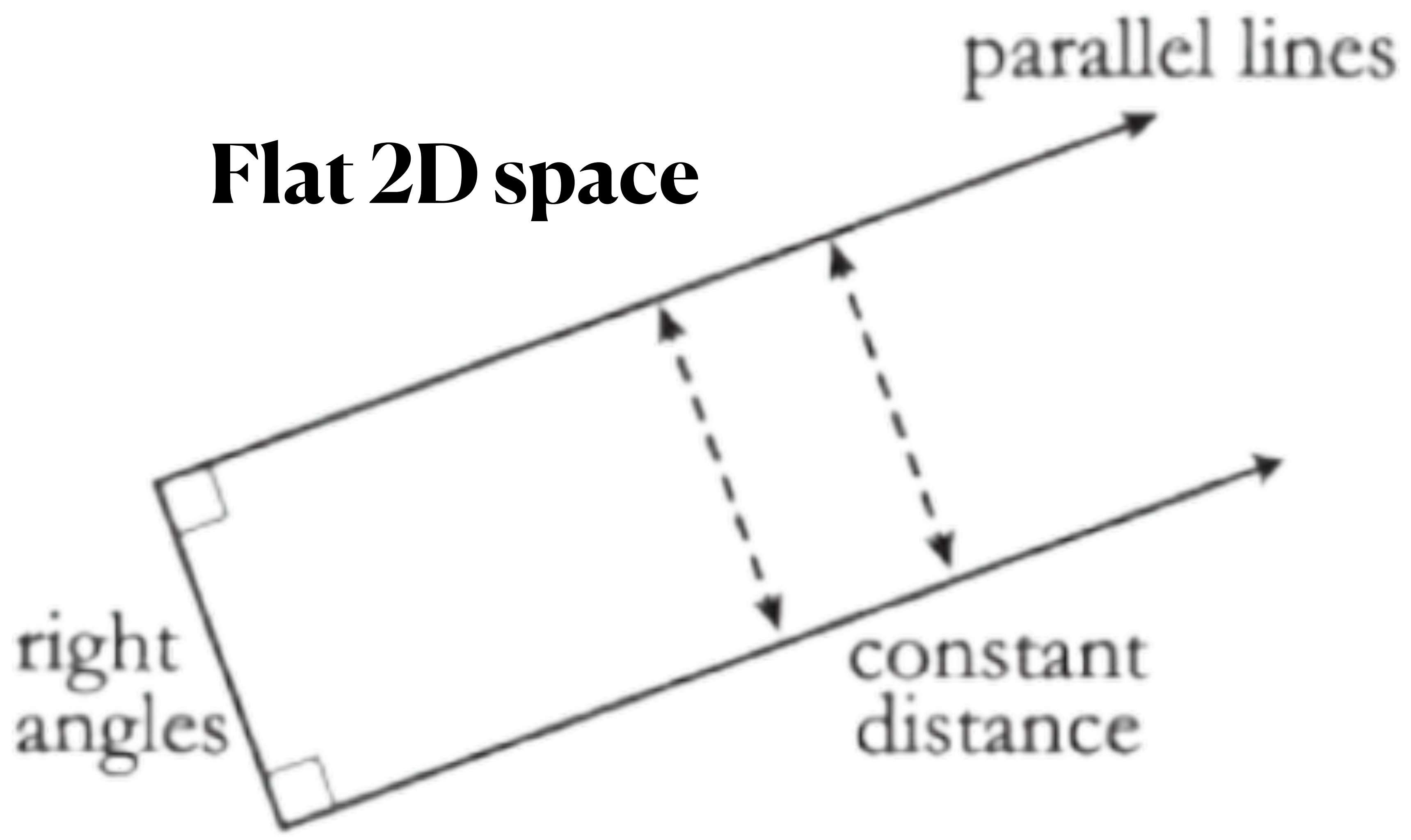


Parallel Axiom:

5. That, if a straight line falling on two straight lines, make the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than two right angles.



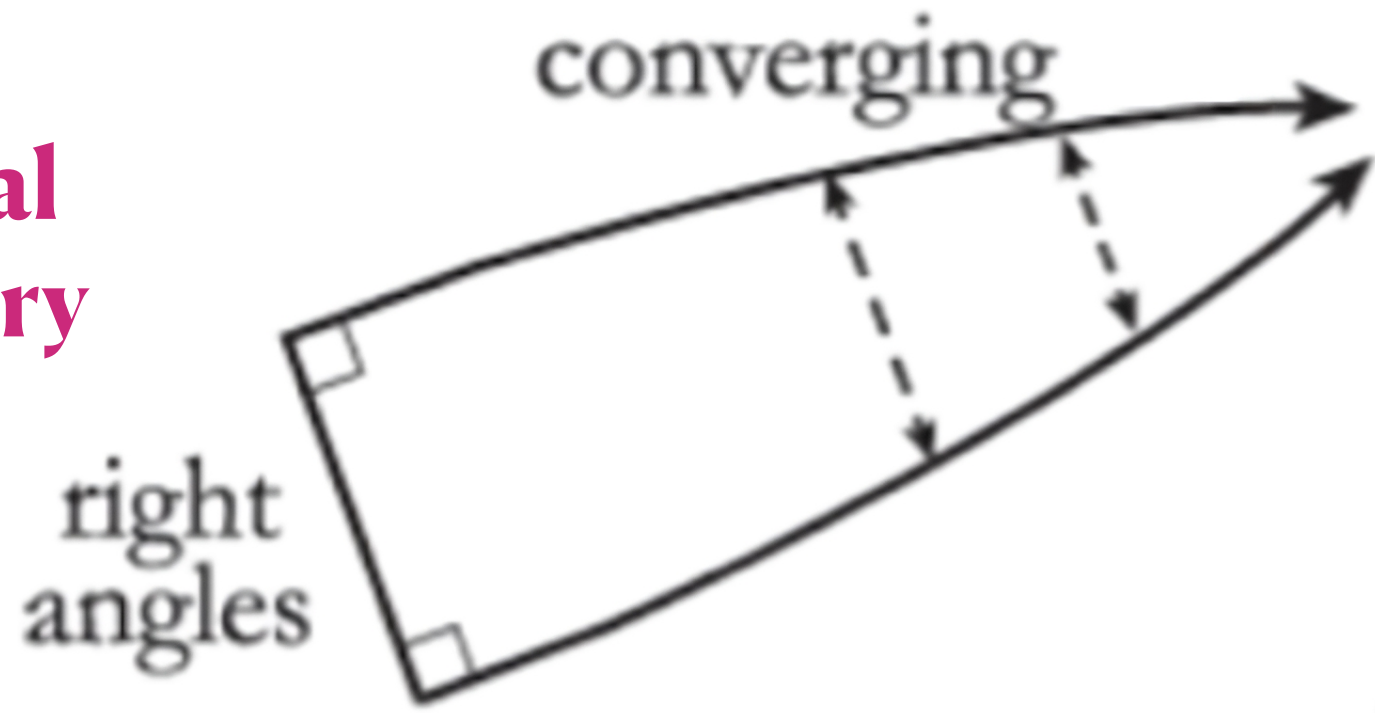
Flat 2D space



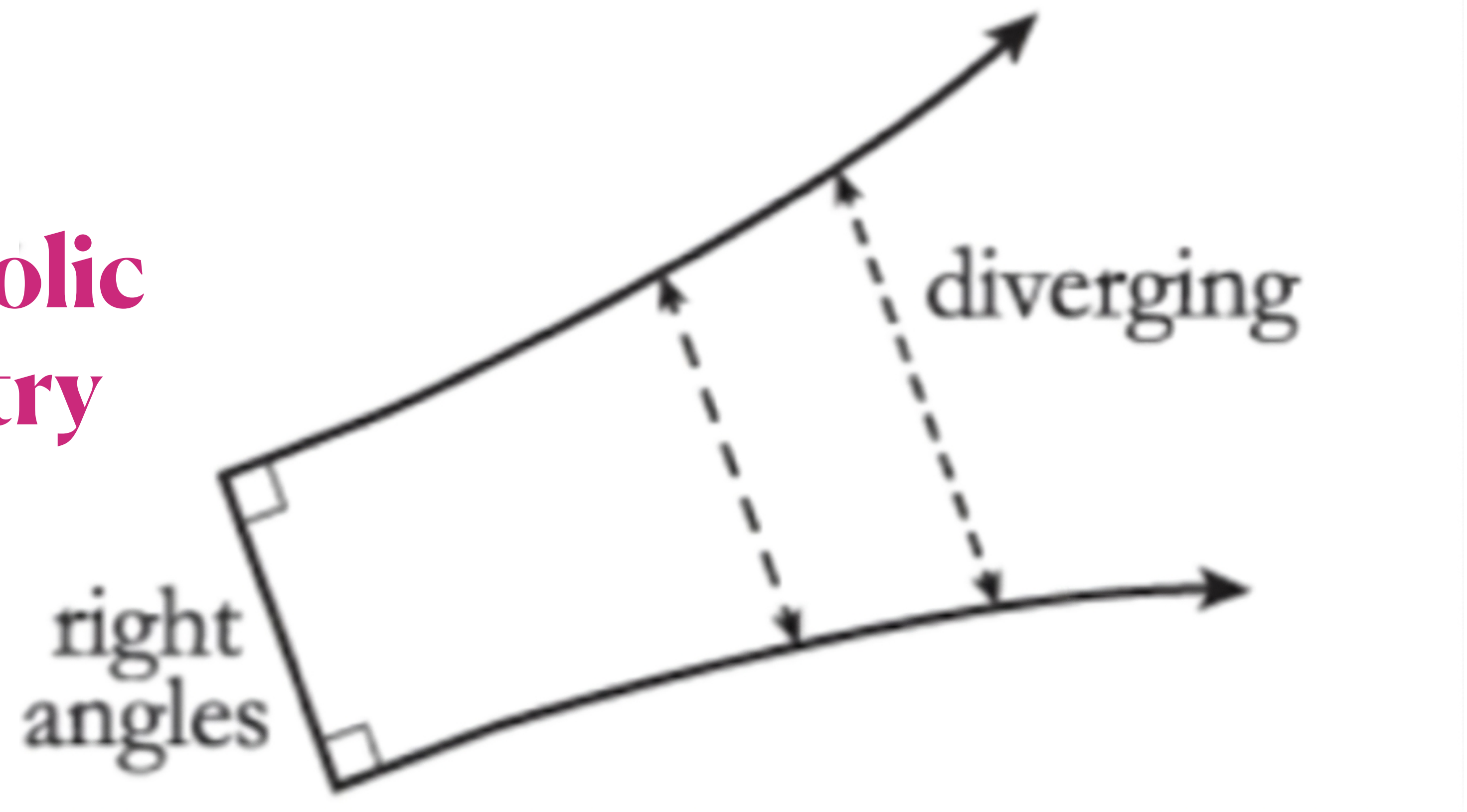
The emergence of non-Euclidean Geometry

1. **Euclid and the Parallel Axiom:**
   - Axiom: Only one non-intersecting line exists through a point not on a given line.
   - Recognized as less obvious and not derivable from other axioms.
2. **19th-Century Breakthroughs:**
   - Development of geometries that reject the parallel axiom.

Spherical Geometry



Hyperbolic Geometry



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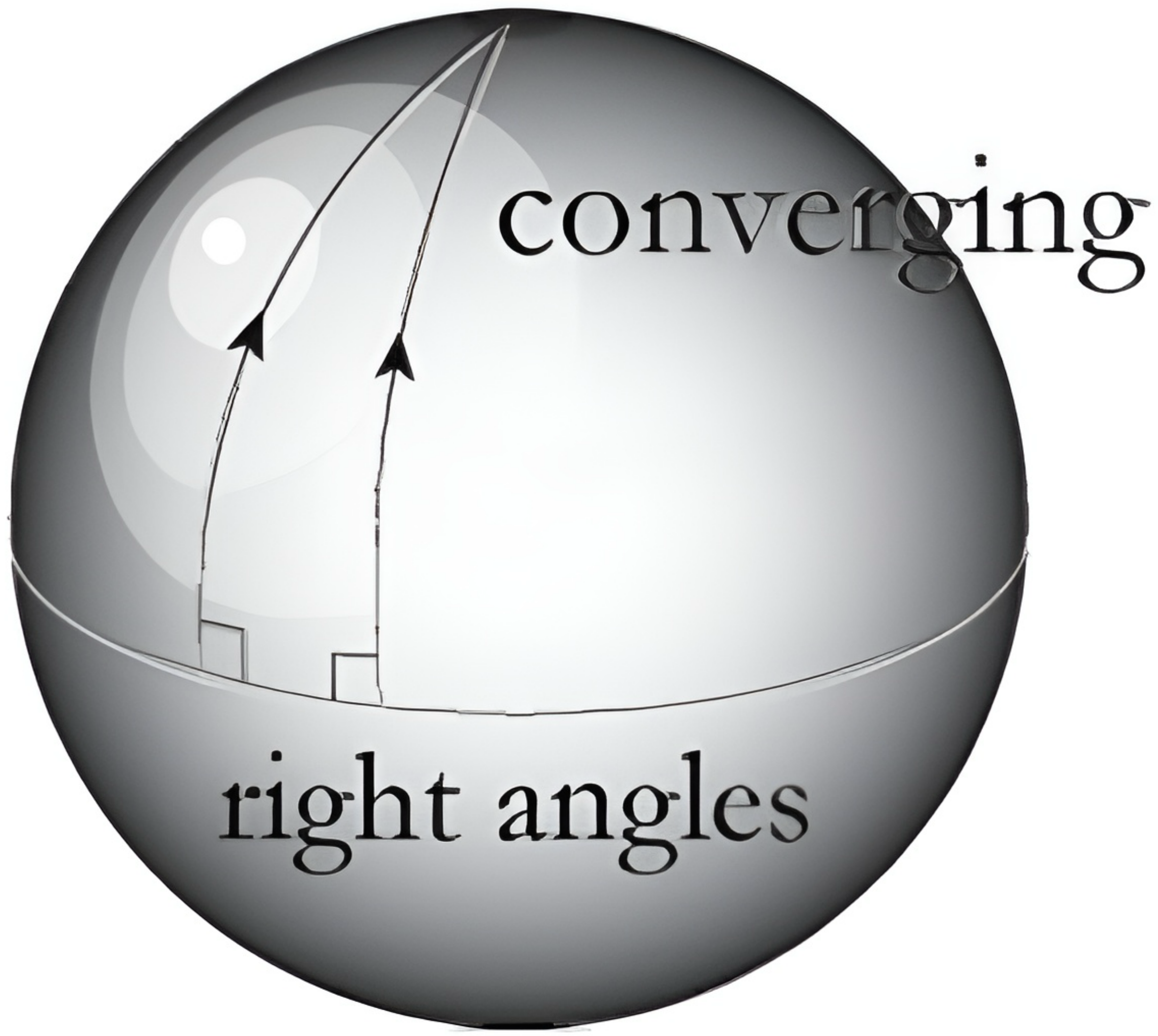

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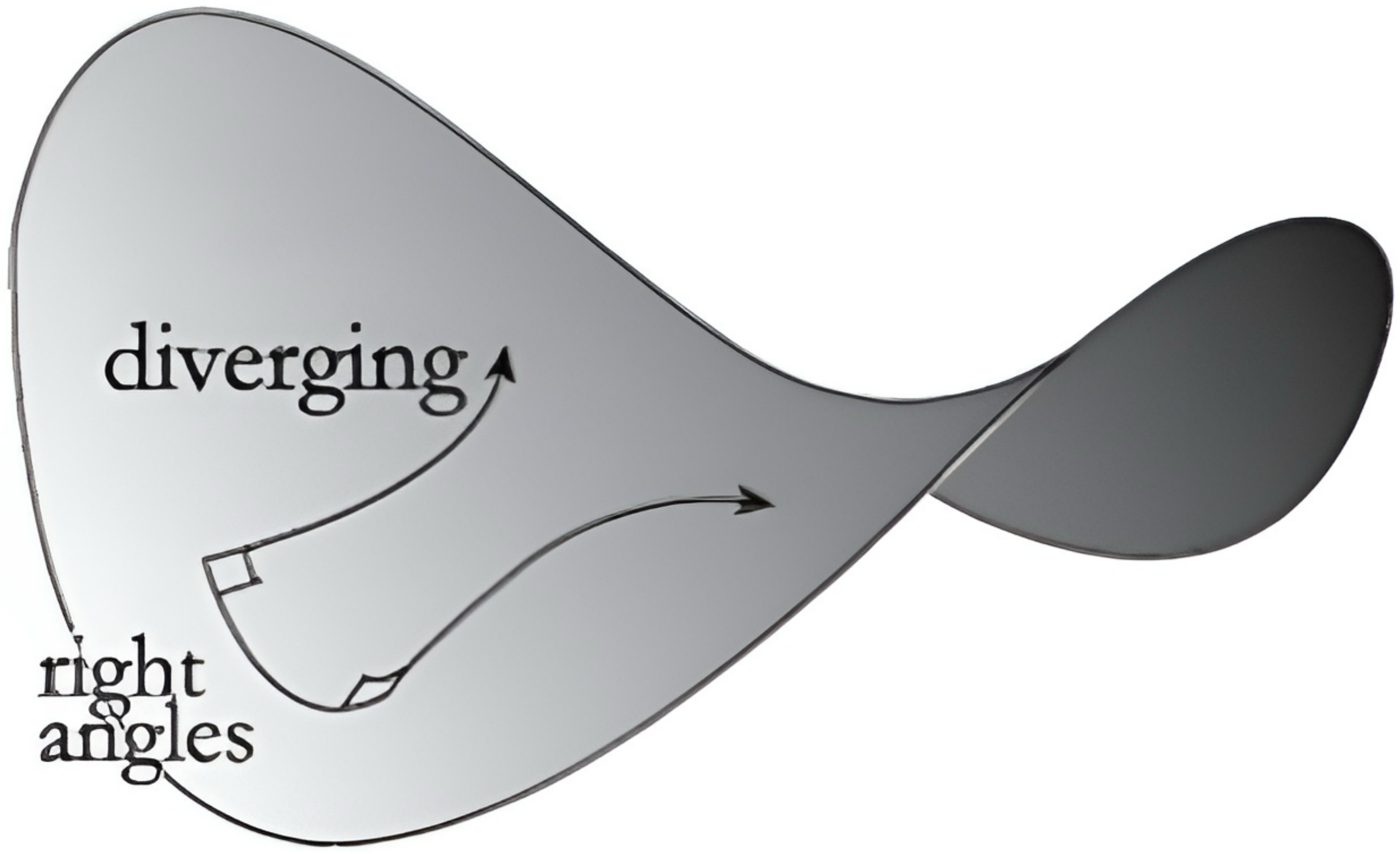
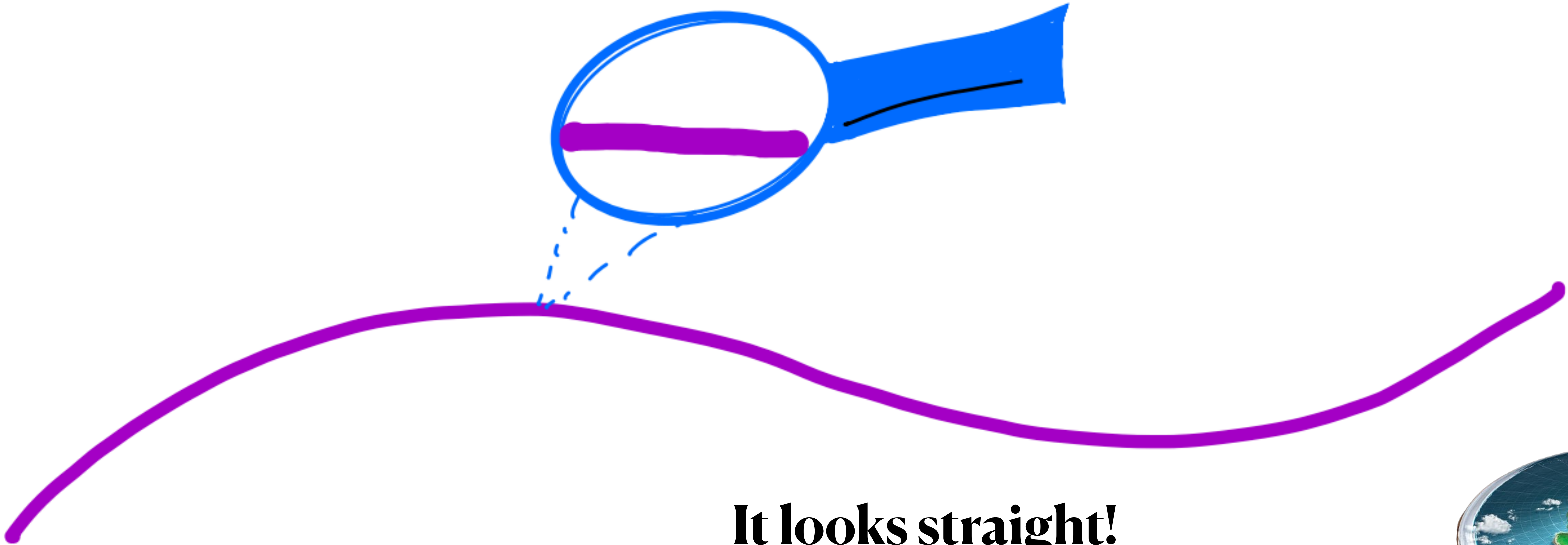


But what if we zoom in a lot?



It looks straight!

(… and that’s why flat-Earthers!)

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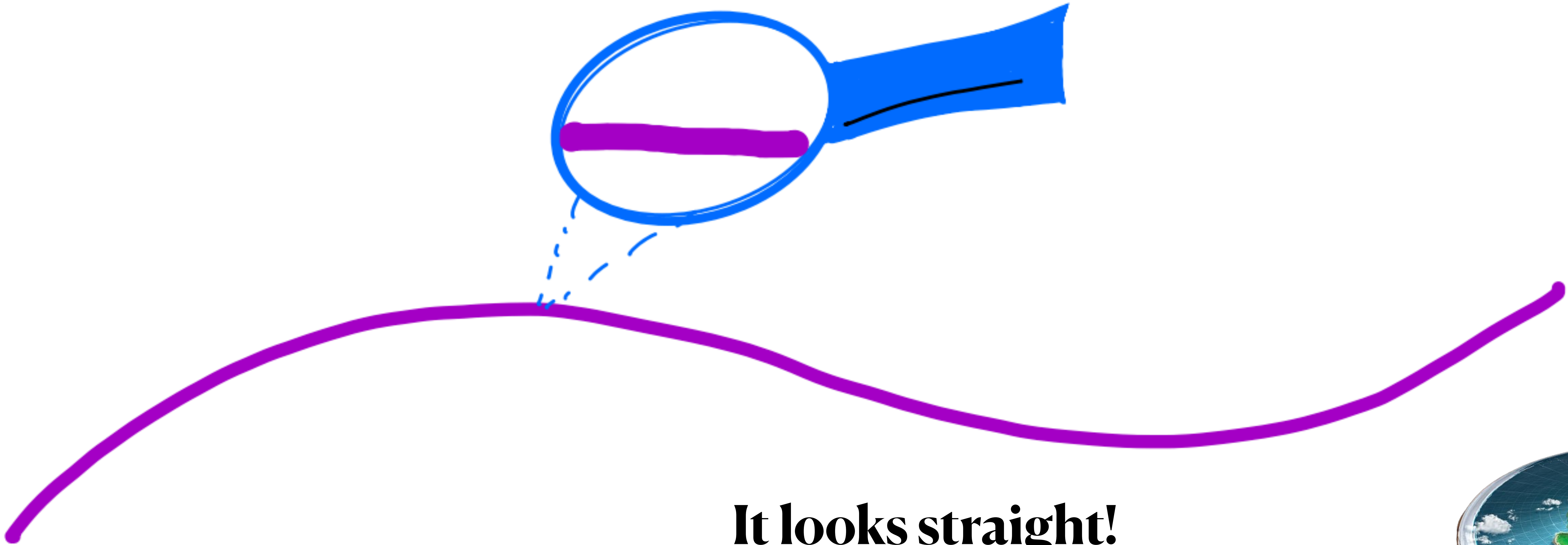

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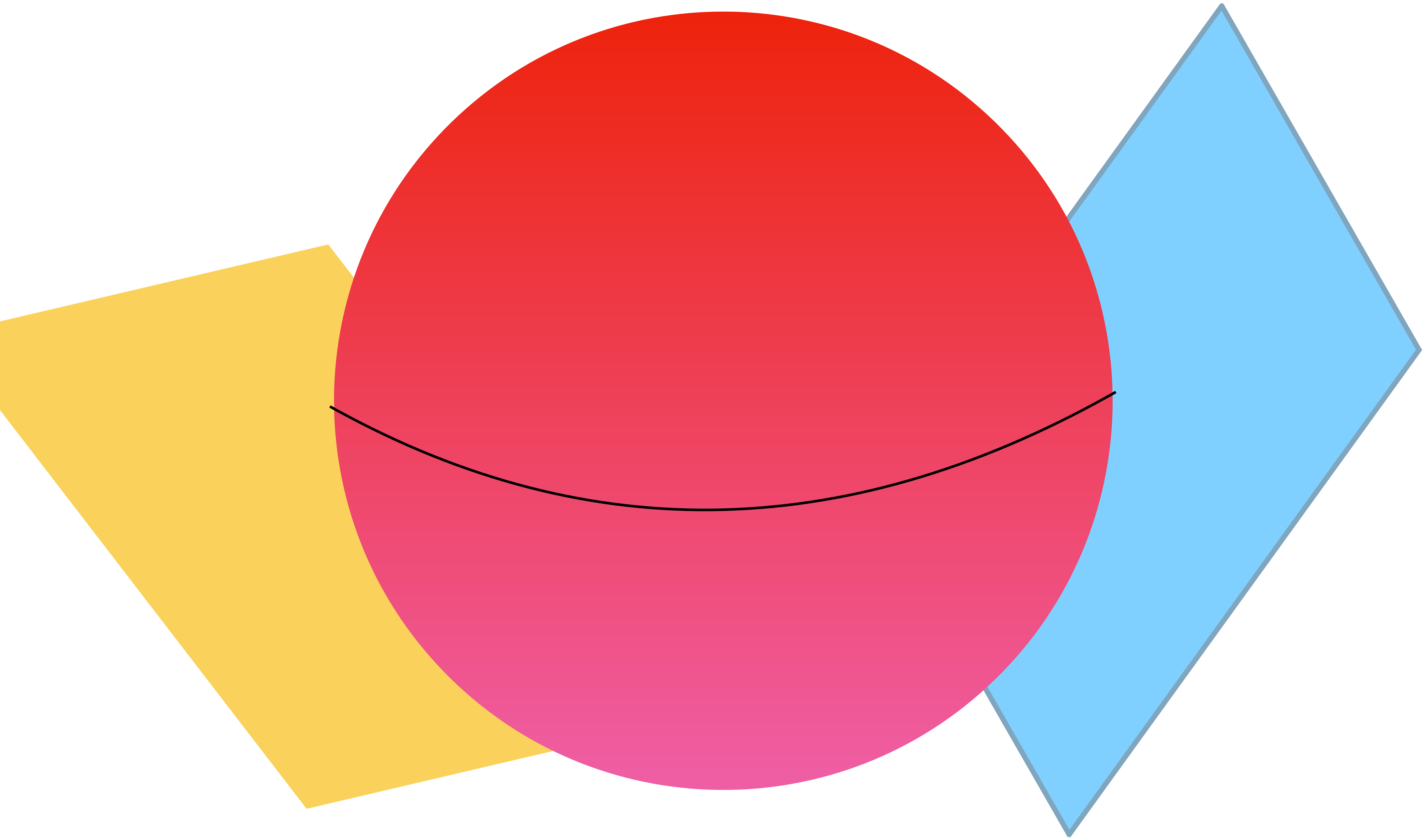
Manifold: An infinite set of points smoothly connected into a space of some definite dimensionality that looks flat!



- What if the sphere was a plane?

- How do we connect the planes?

A new perspective on Mathematics

1. **Shift in Mathematical Understanding:**
   - Mathematics goes beyond intuition
   - Focuses on logical inference from assumptions, irrespective of intuition

2. **Implications for Physics:**
   - Mathematics in physics can lead to counterintuitive results
   - Example: Non-Euclidean geometry hinted that physical reality might surpass intuitive understanding

Physics and the new mathematical paradigm

1. **Einstein and Special Relativity:**
   - Marks a shift to counterintuitive physics
   - Introduces a new understanding of space and time (not yet involving curved space)

2. **Legacy of Non-Euclidean Ideas:**
   - Paves the way for general relativity and other advancements
   - Physics increasingly departs from reliance on intuitive concepts

But how?!

Inertia and Gravity

- we “measure" spatial and temporal displacements/relations
- from Newton’s laws:

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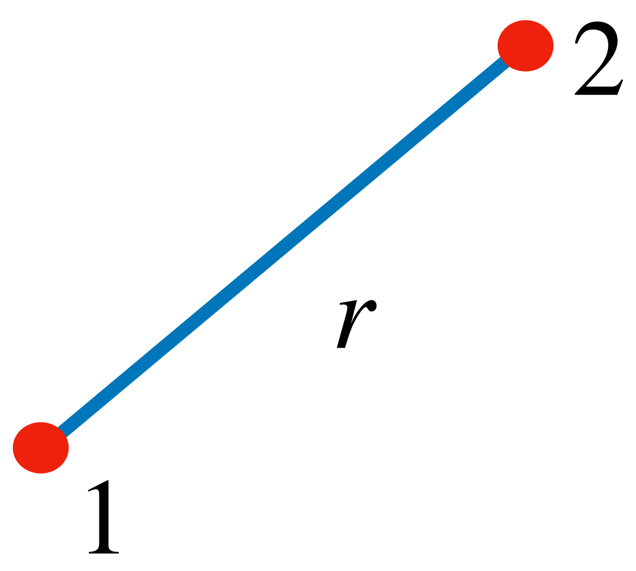
Thus, $x_2 - x_1 = \frac{1}{2} a \times t^2$!

No way to measure universal uniform acceleration differences!

- gravity is peculiar because

$$F_g = G\frac{M_1 M_2}{r^2}$$



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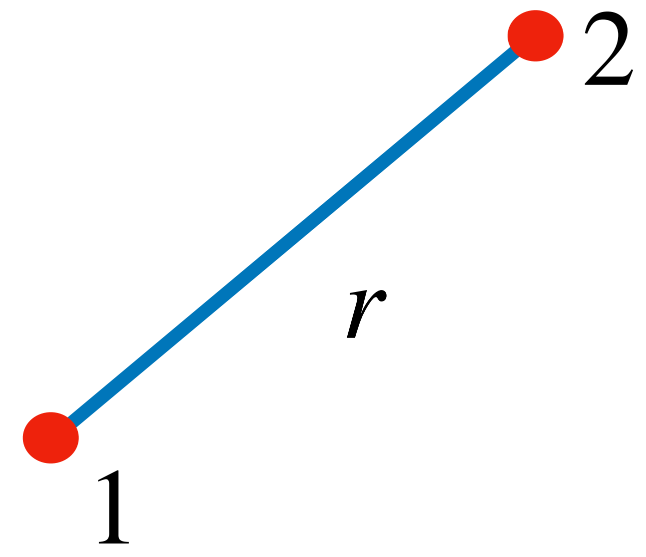

but

$$F_{1\to 2} = m_2 a_{1\to 2} \quad \Rightarrow \quad a_{1\to 2} = \frac{F_{1\to 2}}{m_2} = G\frac{M_1}{r^2}\ \cancel{\frac{M_2}{m_2}} = 1$$

$M_2 = m_2$ to the best of my our empirical tests!

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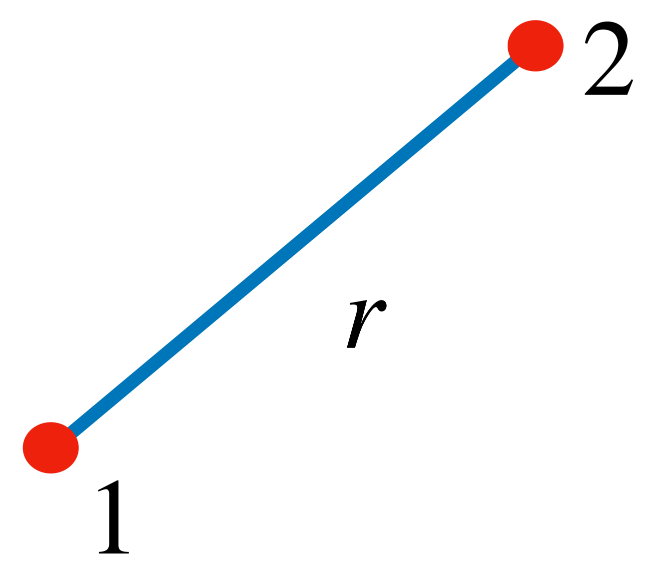

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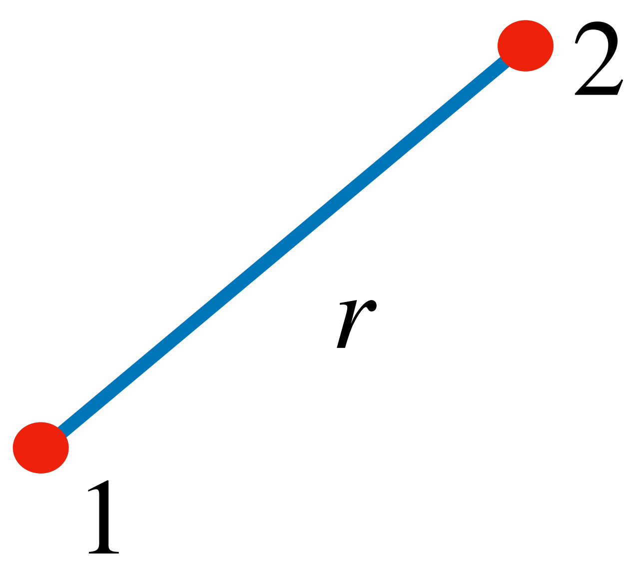

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- electromagnetism: $$F = k\frac{e_1 e_2}{r^2} \quad \Rightarrow \quad a_{1\to 2} = k\frac{e_1}{r^2}\frac{e_2}{m_2}$$

It does **not** cancel!

So, not only does (uniform) gravity seem to impart acceleration on a probe independently of its composition (e.g. mass), but it also cannot be distinguished from uniform acceleration.

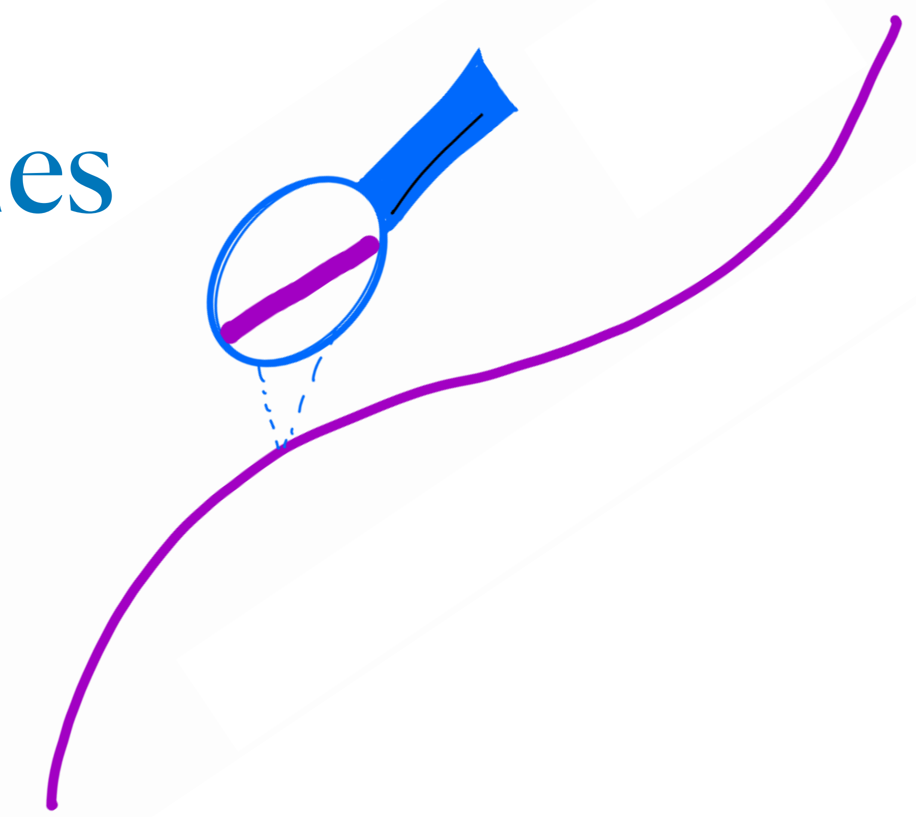
- As uniform velocity was undetectable, so are local (uniform) gravitational fields!

- **Thus, we seek to formulate physical laws that hold for any frame that is falling freely under gravity**

    - Universal force is not force at all, since they were defined by causing deviation from inertial motion. But now gravity itself defines inertial frames

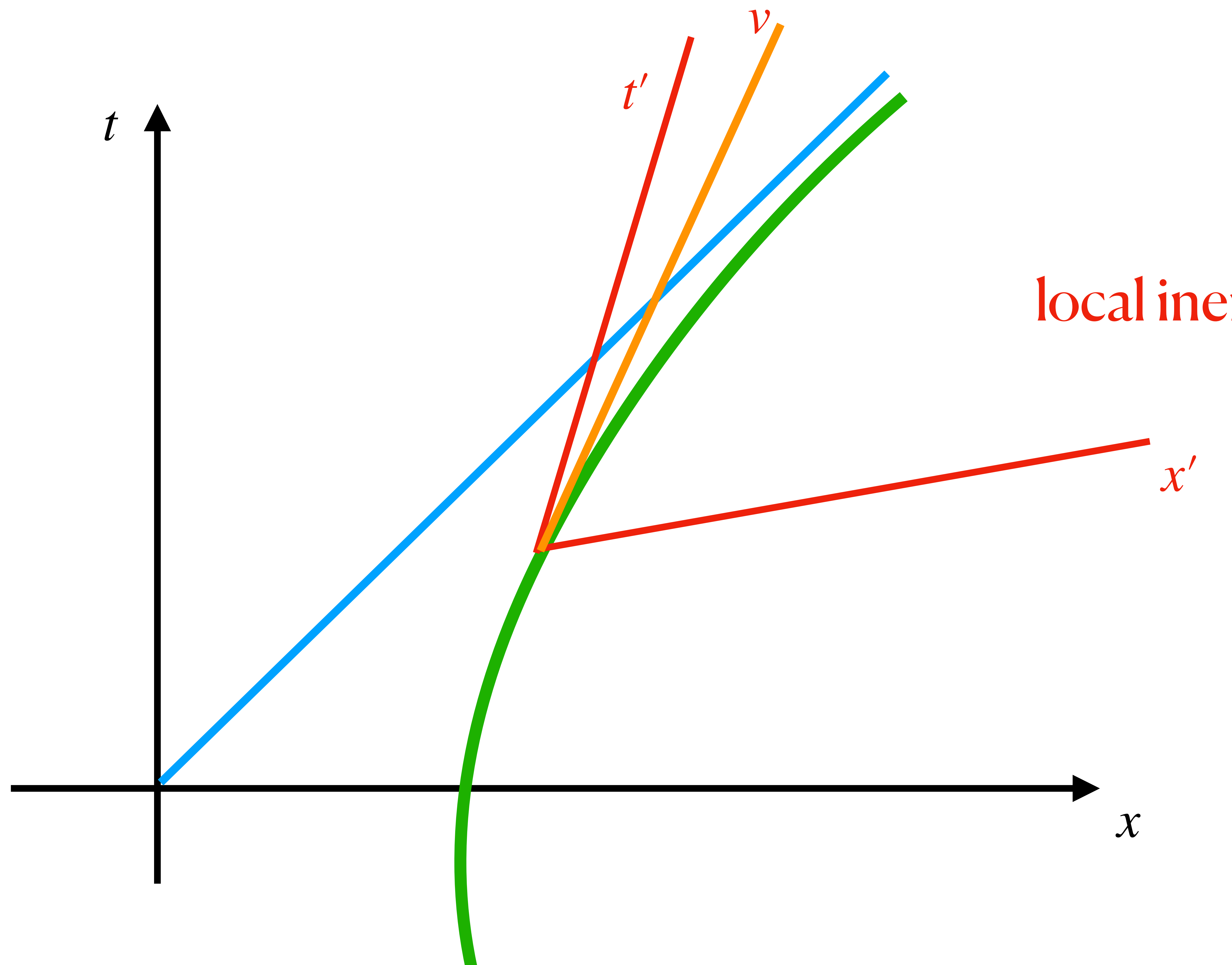
    - It breaks the idea that inertial structure is background structure, since gravity is determined by mass distribution

    - Inertial structure is local instead of fixed for the whole Universe



Principle of : Equivalence

In small regions of spacetime, the effects of gravitation are equivalent to those of being in an accelerating frame of reference. Moreover, these frames are a collection of inertial frames, so the laws of physics reduce to those of special relativity.



local inertial structure!

For larger regions of spacetime, when curvature is relevant, we see the difference!

But we used Newton's equations to make these arguments!

- Newton's gravity depends on particle's mass

- But we saw that in special relativity that has to do only with the rest energy, but what about kinetic forms of energy? We could change the frame of reference so that we sit on top of the particle, but then no kinetic energy in that frame? It cannot be frame-dependent!

Bottom line: Newton gravity is incompatible with special relativity!

Einstein's Equations

the energy-momentum tensor

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

the metric

$$R_{00} - \frac{1}{2} g_{00} R = 8\pi G T_{00}$$

$$R_{01} - \frac{1}{2} g_{01} R = 8\pi G T_{01}$$

$$R_{02} - \frac{1}{2} g_{02} R = 8\pi G T_{02}$$

$$R_{03} - \frac{1}{2} g_{03} R = 8\pi G T_{03}$$

$$R_{11} - \frac{1}{2} g_{11} R = 8\pi G T_{11}$$

$$R_{12} - \frac{1}{2} g_{12} R = 8\pi G T_{12}$$

$$R_{13} - \frac{1}{2} g_{13} R = 8\pi G T_{13}$$

$$R_{22} - \frac{1}{2} g_{22} R = 8\pi G T_{22}$$

$$R_{23} - \frac{1}{2} g_{23} R = 8\pi G T_{23}$$

$$R_{33} - \frac{1}{2} g_{33} R = 8\pi G T_{33}$$

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma}(-\partial_{\sigma} g_{\nu\rho} + \partial_{\nu} g_{\sigma\rho} + \partial_{\rho} g_{\nu\sigma})$$

$$R_{00} - \frac{1}{2} g_{00} R = 8\pi T_{00} \longrightarrow \left(\partial_\rho \Gamma^\rho_{00} - \partial_0 \Gamma^\rho_{\rho 0} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{00} - \Gamma^\rho_{0\lambda}\Gamma^\lambda_{\rho 0}\right) - \frac{1}{2} g_{00}\, g^{\alpha\beta}\left(\partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha}\right) = 8\pi T_{00}$$

$$R_{01} - \frac{1}{2} g_{01} R = 8\pi T_{01} \longrightarrow \left(\partial_\rho \Gamma^\rho_{01} - \partial_2 \Gamma^\rho_{\rho 0} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{10} - \Gamma^\rho_{3\lambda}\Gamma^\lambda_{\rho 0}\right) - \frac{1}{2} g_{03}\, g^{\alpha\beta}\left(\partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha}\right) = 8\pi T_{03}$$

$$R_{02} - \frac{1}{2} g_{02} R = 8\pi T_{02} \longrightarrow \left(\partial_\rho \Gamma^\rho_{02} - \partial_2 \Gamma^\rho_{\rho 0} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{30} - \Gamma^\rho_{3\lambda}\Gamma^\lambda_{\rho 0}\right) - \frac{1}{2} g_{03}\, g^{\alpha\beta}\left(\partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha}\right) = 8\pi T_{03}$$

$$R_{03} - \frac{1}{2} g_{03} R = 8\pi T_{03} \longrightarrow \left(\partial_\rho \Gamma^\rho_{03} - \partial_3 \Gamma^\rho_{\rho 0} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{30} - \Gamma^\rho_{3\lambda}\Gamma^\lambda_{\rho 0}\right) - \frac{1}{2} g_{03}\, g^{\alpha\beta}\left(\partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha}\right) = 8\pi T_{03}$$

$$R_{13} - \frac{1}{2} g_{11} R = 8\pi T_{11} \longrightarrow \left(\partial_\rho \Gamma^\rho_{11} - \partial_2 \Gamma^\rho_{\rho 1} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{31} - \Gamma^\rho_{3\lambda}\Gamma^\lambda_{\rho 1}\right) - \frac{1}{2} g_{13}\, g^{\alpha\beta}\left(\partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha}\right) = 8\pi T_{13}$$

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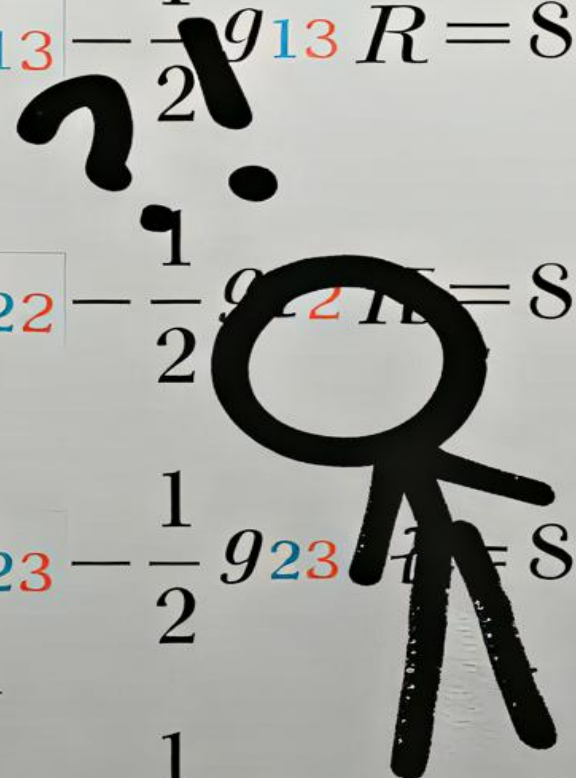

$$R_{22} - \frac{1}{2} g_{22} R = 8\pi T_{22} \longrightarrow \left(\partial_\rho \Gamma^\rho_{22} - \partial_2 \Gamma^\rho_{\rho 2} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{32} - \Gamma^\rho_{3\lambda}\Gamma^\lambda_{\rho 2}\right) - \frac{1}{2} g_{23}\, g^{\alpha\beta}\left(\partial_\rho \Gamma^\rho_{\beta\alpha} - \partial_\beta \Gamma^\rho_{\rho\alpha} + \Gamma^\rho_{\rho\lambda}\Gamma^\lambda_{\beta\alpha} - \Gamma^\rho_{\beta\lambda}\Gamma^\lambda_{\rho\alpha}\right) = 8\pi T_{33}$$

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What is the metric?

$$\tau^2 = \Delta t^2 - \Delta x^2 - \Delta y^2 - \Delta z^2$$

$$g_{\mu\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

(Minkowski flat spacetime)

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(Minkowski flat spacetime)

$$\downarrow$$

$$\tau^2 = g_{tt}\Delta t^2 + 2g_{tx}\Delta t\Delta x + g_{xx}\Delta x^2 + \ldots$$

$$g_{\mu\nu} = \begin{pmatrix} g_{tt} & g_{tx} & g_{ty} & g_{tz} \\ g_{tx} & g_{xx} & g_{xy} & g_{xz} \\ g_{ty} & g_{xy} & g_{yy} & g_{yz} \\ g_{tz} & g_{xz} & g_{yz} & g_{zz} \end{pmatrix}$$

The information contained in the metric allows us to talk about the geometrical properties of any shape in that geometry, which encodes the properties of the space itself!

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu} + g_{\mu\nu}\Lambda$$

It predicts correctly the evolution of the Universe, the existence of black holes, the propagation of gravitational waves, and other phenomena that Einstein had no idea at the time.

What to make of it?

Cosmology: proposal- the Universe is homogeneous and isotropic at large scales (generalization of Copernican principle!)

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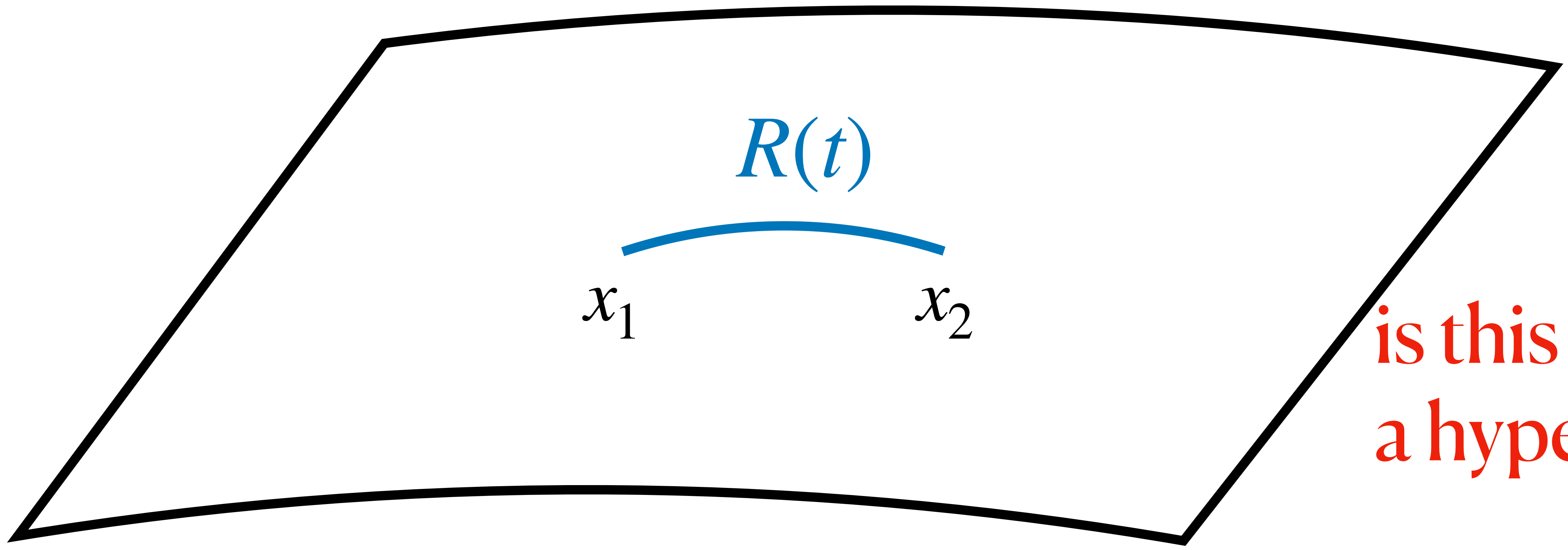
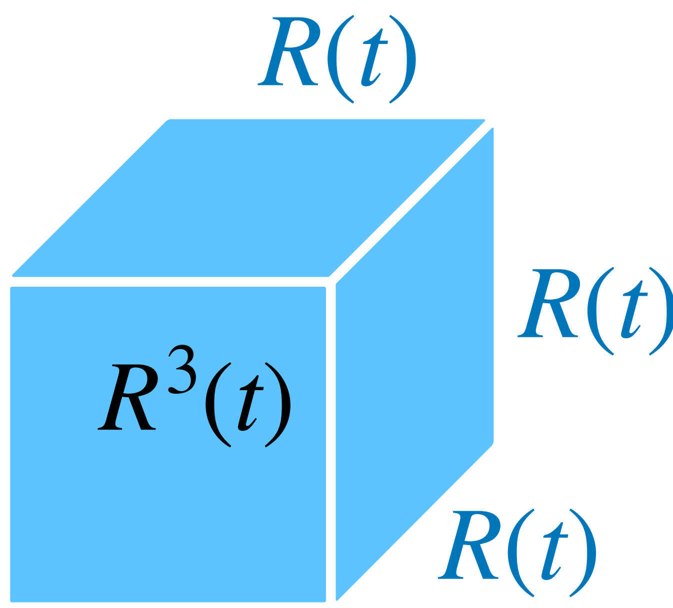
proposal- the Universe is homogeneous and isotropic at large scales (generalization of Copernican principle!)

- metric:



is this a place? a sphere? a hyperboloid?

volume



Cosmology:

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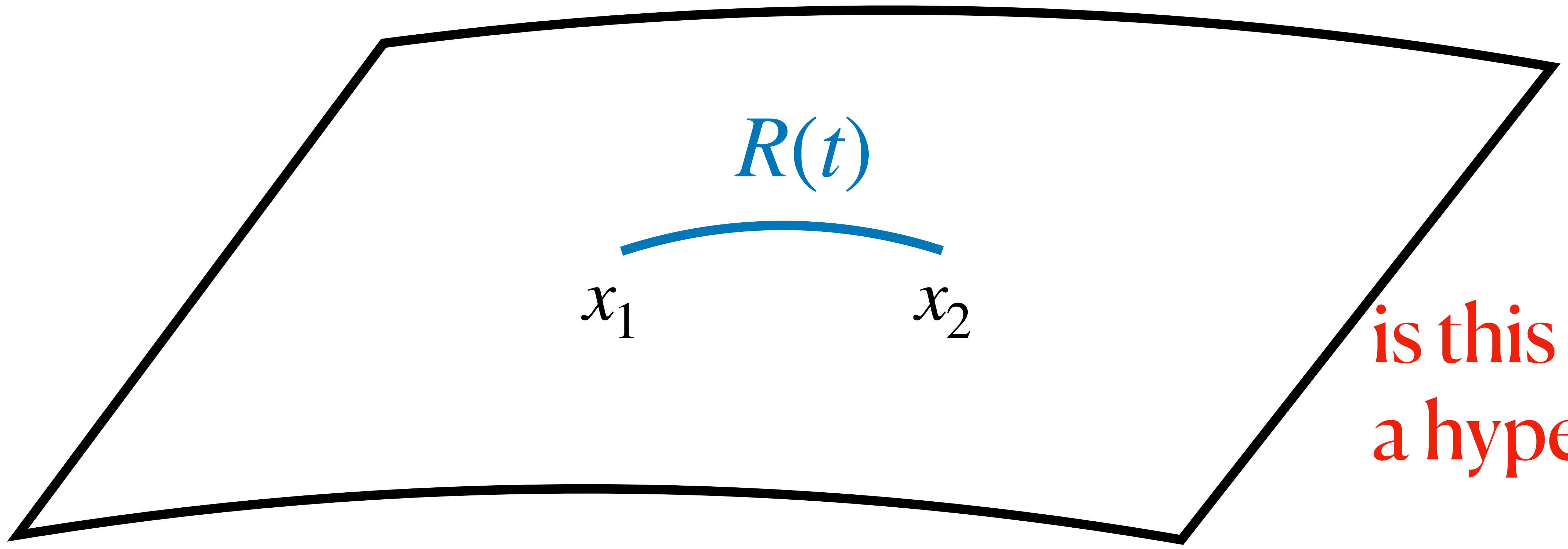

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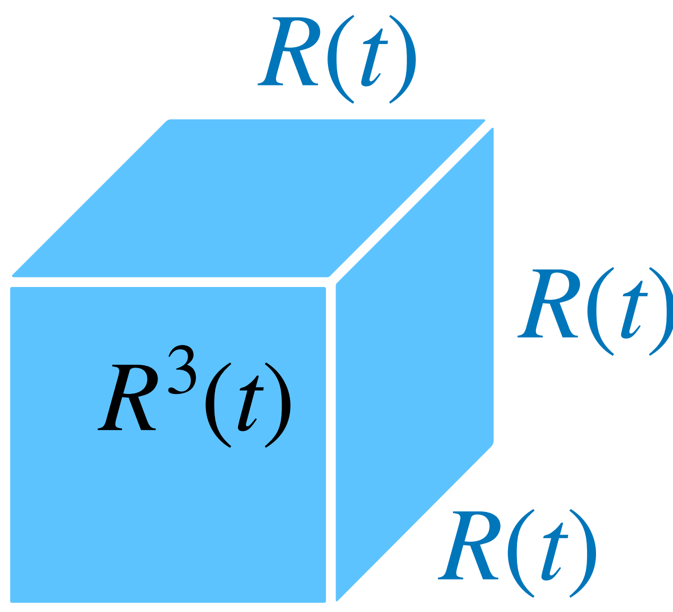

- homogeneous and isotropic energy-matter: FLUID
  - Pressure: $p(t)$
  - Energy density: $\rho(t) = \dfrac{\text{Energy}}{\text{volume}}$

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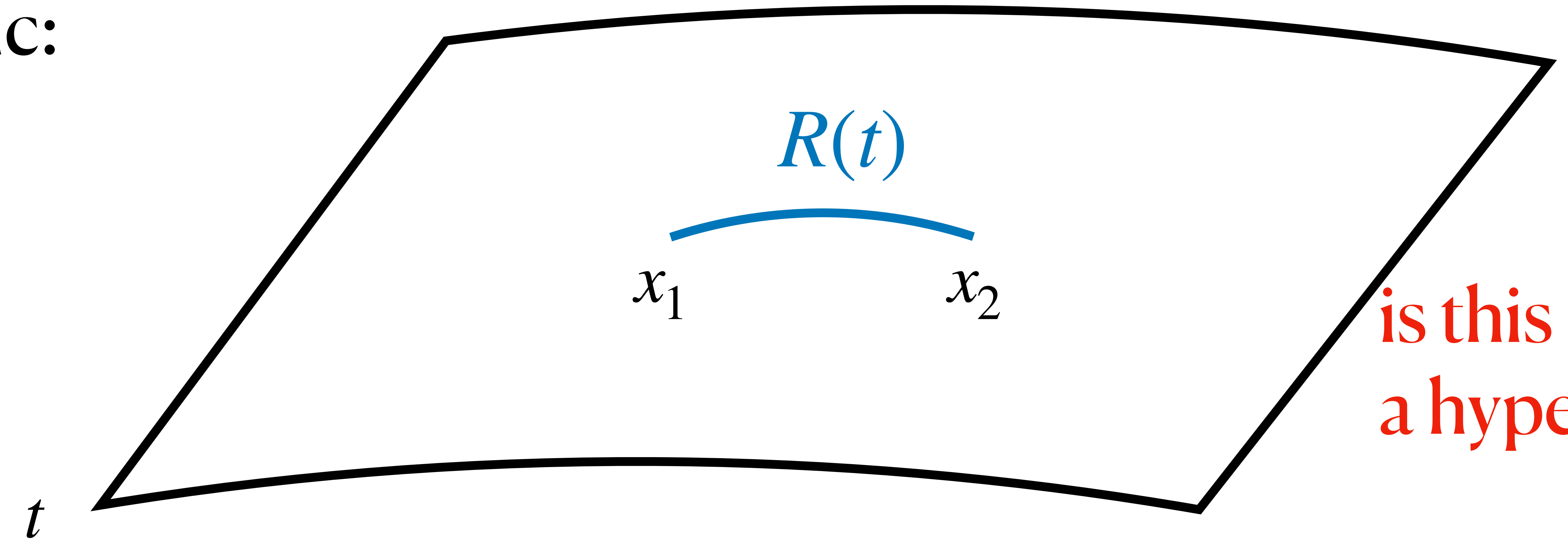

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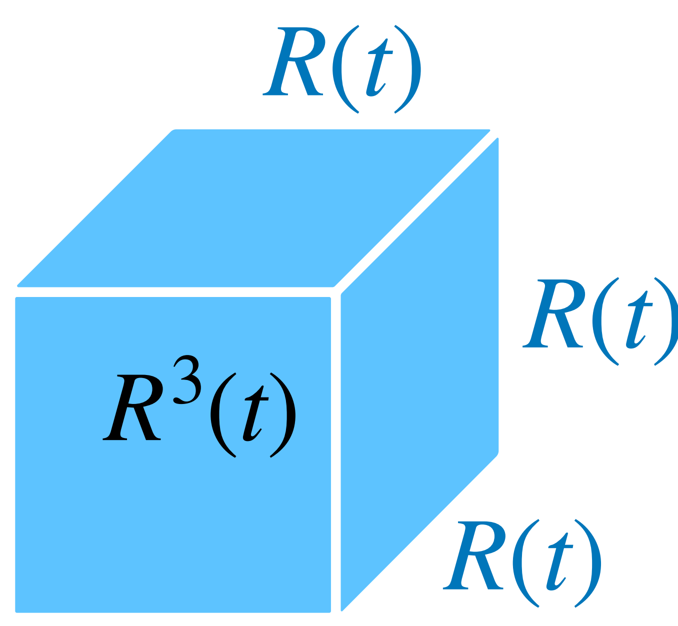

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Friedmann equations:

$$\left(\frac{\dot{R}(t)}{R(t)}\right)^2 + \frac{k}{R(t)^2} = \frac{8\pi G}{3}\rho(t) + \frac{\Lambda}{3}$$

$$\frac{\ddot{R}(t)}{R(t)} = -\frac{4\pi G}{3}\left[\rho(t) + 3\,\frac{p(t)}{c^2}\right] + \frac{\Lambda}{3}$$

$(k = 0)$
plane

- continuity equation: $$\dot{\rho}(t) + 3\frac{\dot{R}(t)}{R(t)} \times (\rho(t) + p(t)) = 0$$

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  - radiation: $$\rho(t) \propto \frac{1}{R^4(t)}, \qquad R(t) \propto t^{1/2}$$

<u>Energy Conservation</u>

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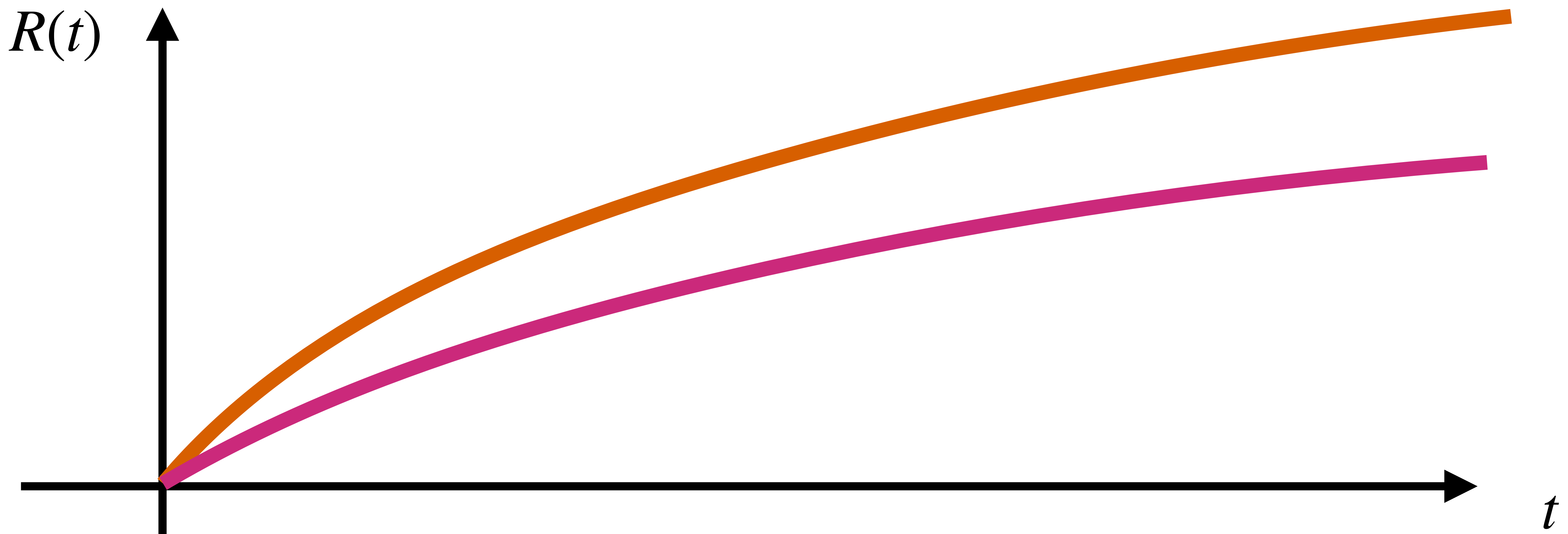
<u>Energy Conservation</u>

✗

✓



- what happens at $t \to \infty$?

$(k=0)$
plane

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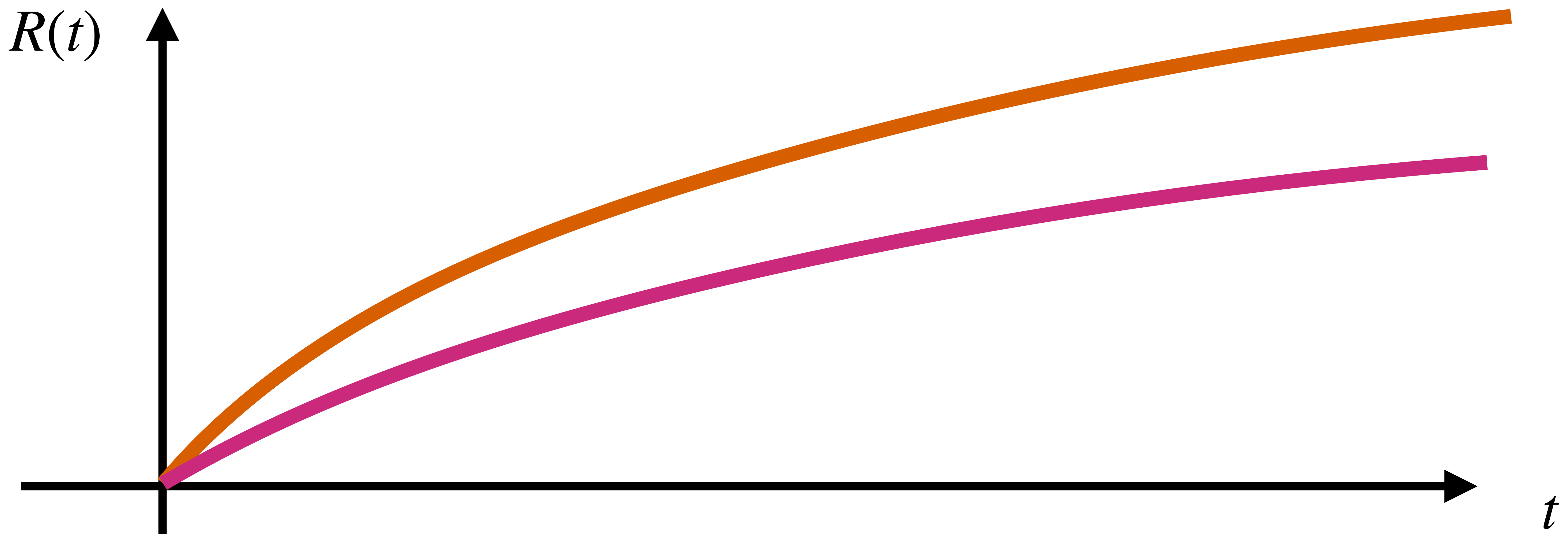
Energy Conservation



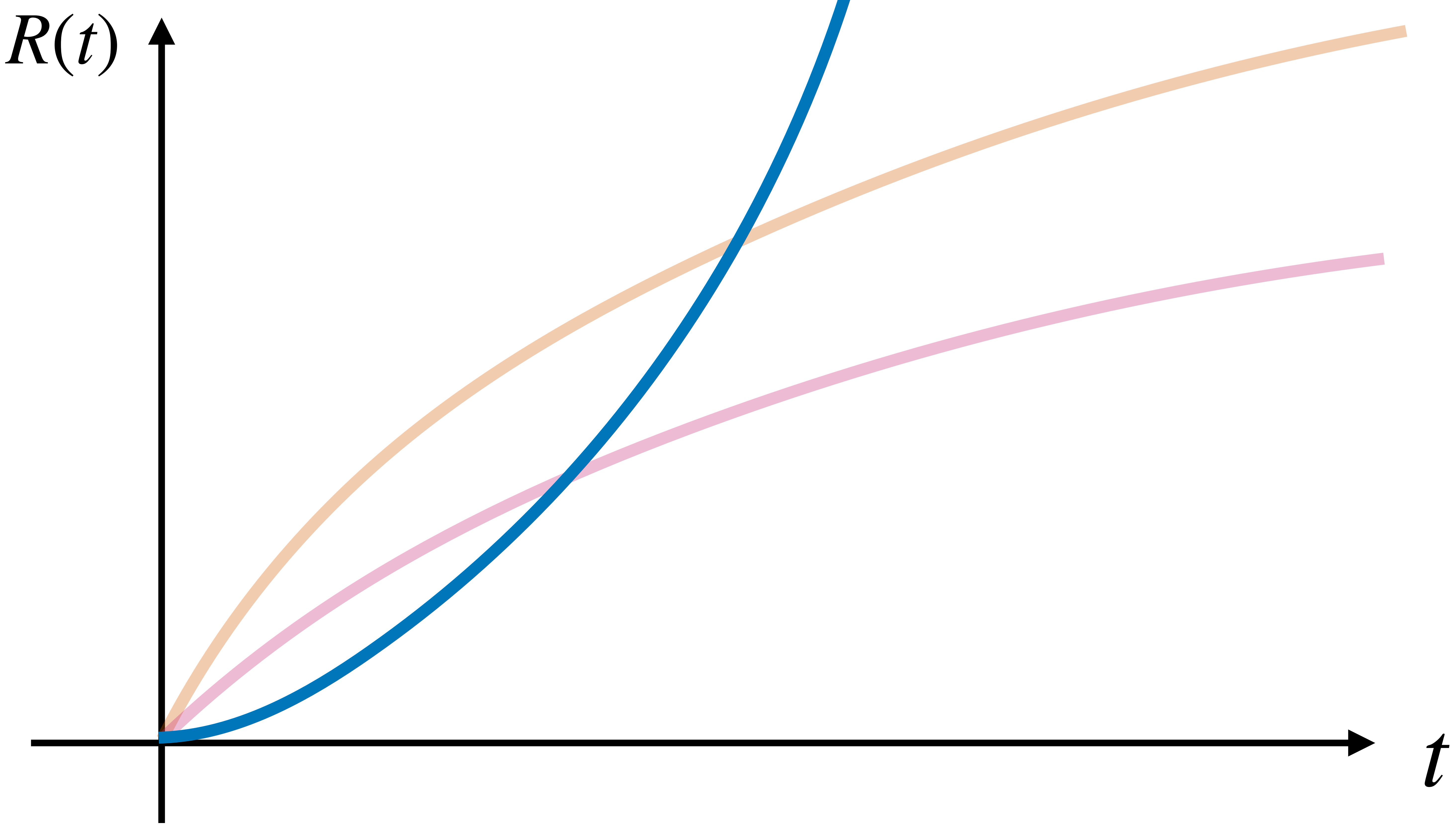
- what happens at $t \to \infty$?
- what happens at $t \to 0$?

- cosmological constant/dark energy:

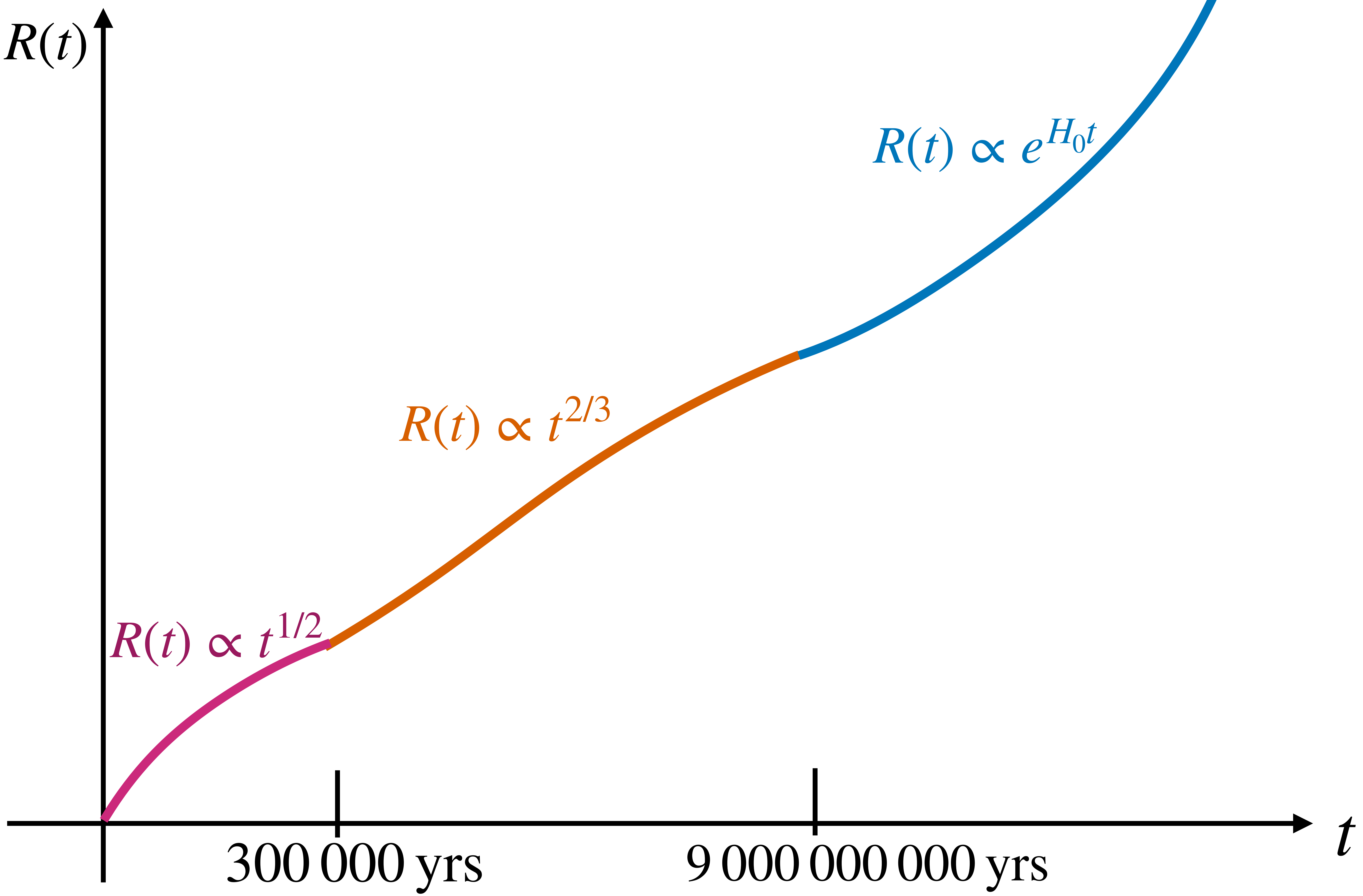
$\rho(t) = \text{constant}$

$R(t) \propto e^{H_0 t}$

Energy Conservation ✗



Our Universe's History



Dark Energy
Accelerated Expansion
Afterglow Light
Pattern
375,000 yrs.
Dark Ages
Development of
Galaxies, Planets, etc.
Inflation
Quantum
Fluctuations
1st Stars
about 400 million yrs.
Big Bang Expansion
13.77 billion years